A GENERALIZED n-PORT CASCADE CONNECTION

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ABSTRACT

An extension of the common two-port cascade connection to a generalized n-port cascade connection is described. An efficient matrix algorithm is then derived which computes the S-parameters of the resulting network from the S-parameters of the two connected networks.

Introduction

This paper describes an extension of the common two-port cascade connection to a generalized n-port cascade connection. Using this technique, the S-parameters of two arbitrarily sized networks, which may be connected together at multiple points, are combined to give the S-parameters of the single resulting network. The connection is illustrated in Figures 1 and 2.

Definition of a Cascade Connection

In this context, a cascade junction may be defined as a connection between two ports, each from a different network. This junction then becomes internal to the resulting combined network, so that no further access is possible. A general cascade connection may consist of many individual cascade junctions between two networks. The only restriction is that at least one port of the resulting network must remain available to the outside world.

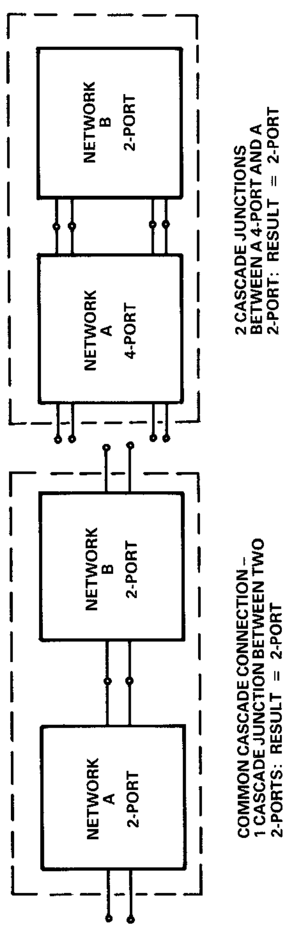

Figure 1 – Typical Cascade Connections

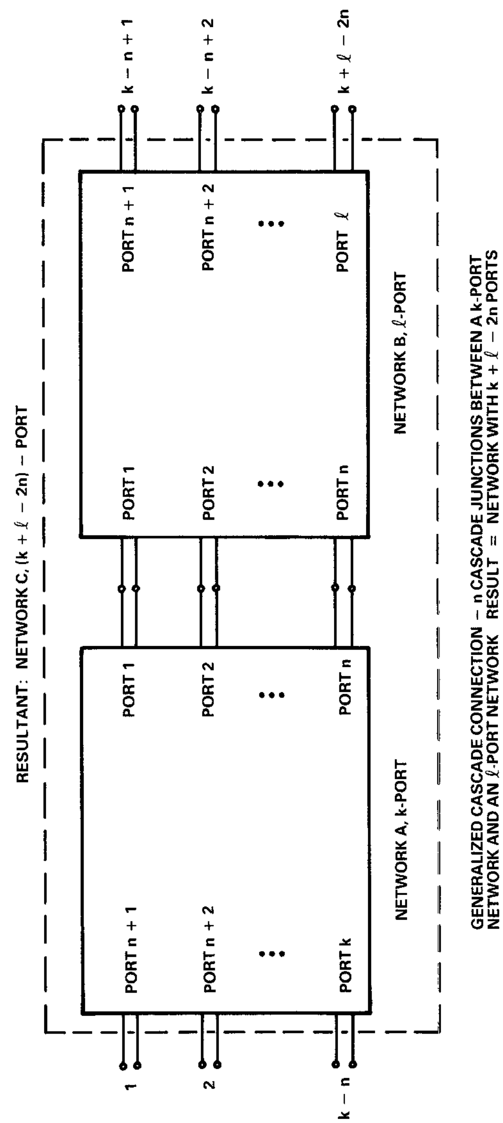

Figure 2 – Generalized Cascade Connection

Theory

Consider the generalized cascade connection shown in Figure 2. There are n cascade junctions, and the port numbers are chosen such that the cascaded ports are the lowest-numbered ones of each network. Network A has k ports, network B has ℓ ports, so the resulting network will have $k + \ell - 2n$ ports. To compute the S-parameter matrix of the resulting network, first partition the S-parameter matrices of networks A and B each into four sections. For network A,

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$$S^A = \left|\begin{array}{cccc|ccc} S^A_{11} & S^A_{12} & \cdots & S^A_{1n} & S^A_{1,\,n+1} & \cdots & S^A_{1k} \\ \vdots & & & \vdots & \vdots & & \vdots \\ S^A_{21} & S_1 & & \vdots & \vdots & S_2 & \vdots \\ \vdots & & & \vdots & \vdots & & \vdots \\ S^A_{n1} & \cdots & & S^A_{nn} & S^A_{n,\,n+1} & \cdots & S^A_{nk} \\ \hline S^A_{n+1,\,1} & \cdots & & S^A_{n+1,\,n} & S^A_{n+1,\,n+1} & \cdots & S^A_{n+1,\,k} \\ \vdots & & & \vdots & \vdots & & \vdots \\ \vdots & S_3 & & \vdots & \vdots & S_4 & \vdots \\ \vdots & & & \vdots & \vdots & & \vdots \\ S^A_{k1} & \cdots & & S^A_{kn} & S^A_{k,\,n+1} & \cdots & S^A_{kk} \end{array}\right| . \qquad (1)$$

Similarly, for network B,

$$S^B = \left|\begin{array}{cccc|ccc} S^B_{11} & S^B_{12} & \cdots & S^B_{1n} & S^B_{1,\,n+1} & \cdots & S^B_{1\ell} \\ \vdots & & & \vdots & \vdots & & \vdots \\ S^B_{21} & S_5 & & \vdots & \vdots & S_6 & \vdots \\ \vdots & & & \vdots & \vdots & & \vdots \\ S^B_{n1} & \cdots & & S^B_{nn} & S^B_{n,\,n+1} & \cdots & S^B_{n\ell} \\ \hline S^B_{n+1,\,1} & \cdots & & S^B_{n+1,\,n} & S^B_{n+1,\,n+1} & \cdots & S^B_{n+1,\ell} \\ \vdots & & & \vdots & \vdots & & \vdots \\ \vdots & S_7 & & \vdots & \vdots & S_8 & \vdots \\ \vdots & & & \vdots & \vdots & & \vdots \\ S^B_{\ell 1} & \cdots & & S^B_{\ell n} & S^B_{\ell,\,n+1} & \cdots & S^B_{\ell\ell} \end{array}\right| . \qquad (2)$$

These eight smaller matrices, S_1 through S_8, are now combined in Equations (3) through (6) to give the partitions of the resultant S-parameter matrix:

$$S_I = S_3 S_5 (I - S_1 S_5)^{-1} S_2 + S_4 \qquad (3)$$

$$S_{II} = S_3 S_5 (I - S_1 S_5)^{-1} S_1 S_6 + S_3 S_6 \qquad (4)$$

$$S_{III} = S_7 (I - S_1 S_5)^{-1} S_2 \qquad (5)$$

$$S_{IV} = S_7 (I - S_1 S_5)^{-1} S_1 S_6 + S_8 \quad . \qquad (6)$$

The S-parameters of the resultant, network C, are then

$$S^C = \left|\begin{array}{c|c} S_I & S_{II} \\ \hline S_{III} & S_{IV} \end{array}\right| \quad . \qquad (7)$$

Degenerate Cases

Equations (3) through (7) are general and cover all cases where $k > n$ and $\ell > n$. That leaves only two cases which must be considered:

1. $k = n$ and $\ell > n$ (all ports of network A are cascaded)— In this case S_2, S_3, and S_4 do not exist, which eliminates Equations (3), (4), and (5). The complete S-parameter matrix of the resulting network is given by Equation (6):

$$S^C = S_{IV} \quad . \qquad (8)$$

2. $\ell = n$ and $k > n$ (all ports of network B are cascaded)— In this case S_6, S_7, and S_8 do not exist, which eliminates Equations (4), (5), and (6). The complete S-parameter matrix of the resulting network C is then given by Equation (3):

$$S^C = S_I \quad . \qquad (9)$$

Application

Equations (3) through (6) are readily implemented on a computer. Only one matrix inversion is required, and the order of the inverted matrix is equal to the number of cascade junctions used. Consequently, computation can be rapid even if large networks (in terms of ports) are used. For a single cascade junction (the ordinary two-port connection, for example), the matrix inversion degenerates to an arithmetic inversion.

The usefulness of this algorithm lies in its flexibility to combine arbitrarily sized networks. Networks of 1, 2, 3, and 4 ports are commonly used together in microstrip or stripline circuits, and this provides a convenient analysis method. Because only S-parameters are used, no parameter conversions are needed on measured data or to get the output data into a form usually desired by microwave engineers.

Example

To illustrate the application of this algorithm, consider the computation of an ideal branch-line coupler. The first step is to define basic circuit elements, which in this case will be networks A, B, and C in Figure 3. These are then combined repeatedly to produce intermediate networks D, E, and F. The last combination produces network G, which is a branch-line coupler. The calculated S-parameters of all seven networks are printed in Figure 4. The frequency is normalized to the center frequency of the coupler.

The real value of this algorithm would be more apparent from this example if nonideal elements were used to model a real-world coupler. The ideal parallel tee (network C) could be replaced with a circuit model for a microstrip tee, and could be as simple or complex as needed.

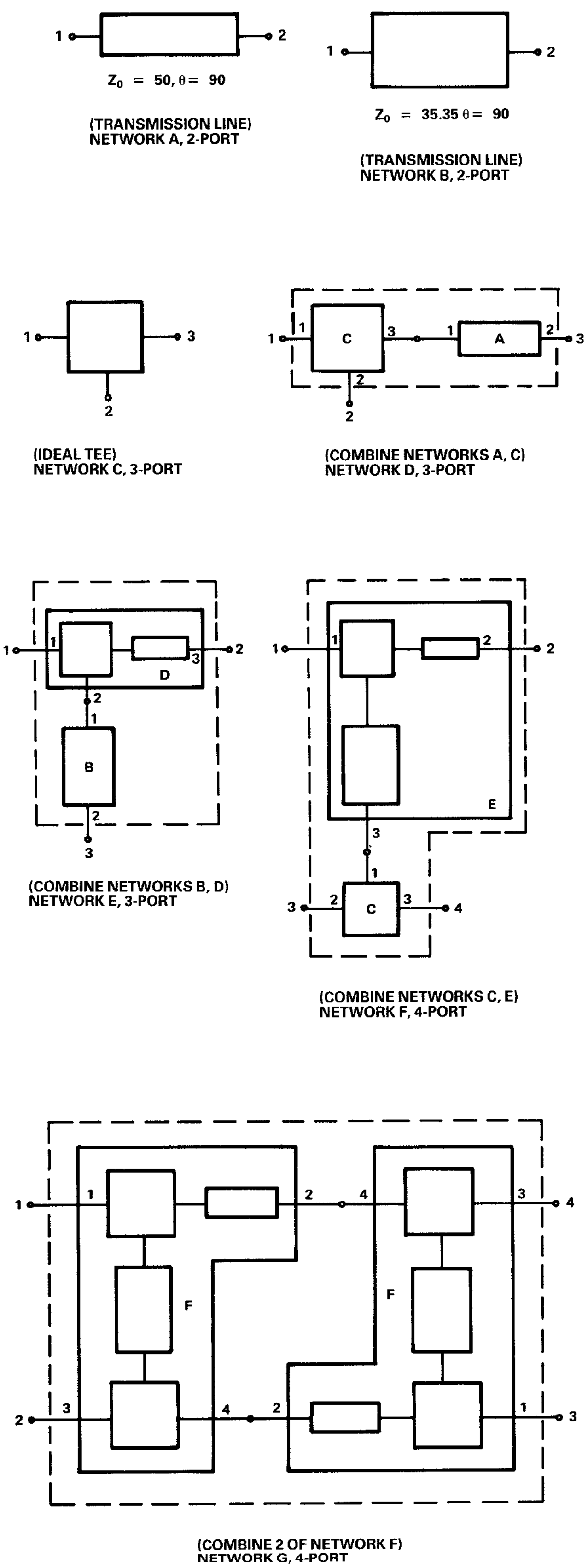

Figure 3 — Connection Sequence for Branch-Line Coupler Computation Example

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BRANCH LINE COUPLER CALCULATION
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     A-S11   A-S11    A-S12   A-S12     A-S21   A-S21     A-S22   A-S22
1.0000   0.000   0.000    1.000 -90.000     1.000 -90.000     0.000   0.000
1.0500   0.000 -184.500   1.000 -94.500     1.000 -94.500     0.000 -184.500

BRANCH LINE COUPLER CALCULATION
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     B-S11   B-S11    B-S12   B-S12     B-S21   B-S21     B-S22   B-S22
1.0000   0.333 -180.000   0.943 -90.000     0.943 -90.000     0.333 -180.000
1.0500   0.333 -184.243   0.943 -94.243     0.943 -94.243     0.333 -184.243

BRANCH LINE COUPLER CALCULATION
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     C-S11   C-S11    C-S12   C-S12     C-S13   C-S13     C-S21   C-S21
1.0000   0.333 -180.000   0.667   0.000     0.667   0.000     0.667   0.000
1.0500   0.333 -180.000   0.667   0.000     0.667   0.000     0.667   0.000
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     C-S22   C-S22    C-S23   C-S23     C-S31   C-S31     C-S32   C-S32
1.0000   0.333 -180.000   0.667   0.000     0.667   0.000     0.667   0.000
1.0500   0.333 -180.000   0.667   0.000     0.667   0.000     0.667   0.000
FREQ      MAG    PHASE
 GHZ     C-S33   C-S33
1.0000   0.333 -180.000
1.0500   0.333 -180.000

BRANCH LINE COUPLER CALCULATION
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     D-S11   D-S11    D-S12   D-S12     D-S13   D-S13     D-S21   D-S21
1.0000   0.333 -180.000   0.667   0.000     0.667 -90.000     0.667   0.000
1.0500   0.333 -180.000   0.667 -360.000    0.667 -94.500     0.667 -360.000
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     D-S22   D-S22    D-S23   D-S23     D-S31   D-S31     D-S32   D-S32
1.0000   0.333 -180.000   0.667 -90.000     0.667 -90.000     0.667 -90.000
1.0500   0.333 -180.000   0.667 -94.500     0.667 -94.500     0.667 -94.500
FREQ      MAG    PHASE
 GHZ     D-S33   D-S33
1.0000   0.333 -360.000
1.0500   0.333  -9.000

BRANCH LINE COUPLER CALCULATION
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     E-S11   E-S11    E-S12   E-S12     E-S13   E-S13     E-S21   E-S21
1.0000   0.500 -180.000   0.500 -90.000     0.707 -90.000     0.500 -90.000
1.0500   0.499 -181.587   0.501 -92.920     0.707 -94.772     0.501 -92.920
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     E-S22   E-S22    E-S23   E-S23     E-S31   E-S31     E-S32   E-S32
1.0000   0.500 -360.000   0.707 -180.000    0.707 -90.000     0.707 -180.000
1.0500   0.499 -10.587    0.707 -189.272    0.707 -94.772     0.707 -189.272
FREQ      MAG    PHASE
 GHZ     E-S33   E-S33
1.0000   0.000 -180.006
1.0500   0.028 -95.083

BRANCH LINE COUPLER CALCULATION
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     F-S11   F-S11    F-S12   F-S12     F-S13   F-S13     F-S14   F-S14
1.0000   0.333 -180.000   0.667 -90.000     0.471 -90.000     0.471 -90.000
1.0500   0.335 -177.894   0.666 -95.558     0.472 -94.244     0.472 -94.244
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     F-S21   F-S21    F-S22   F-S22     F-S23   F-S23     F-S24   F-S24
1.0000   0.667 -90.000    0.333 -360.000    0.471 -180.000    0.471 -180.000
1.0500   0.666 -95.558    0.335  -6.894     0.472 -188.744    0.472 -188.744
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     F-S31   F-S31    F-S32   F-S32     F-S33   F-S33     F-S34   F-S34
1.0000   0.471 -90.000    0.471 -180.000    0.333 -180.000    0.667 -360.000
1.0500   0.472 -94.244    0.472 -188.744    0.335 -177.894    0.666  -1.058
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     F-S41   F-S41    F-S42   F-S42     F-S43   F-S43     F-S44   F-S44
1.0000   0.471 -90.000    0.471 -180.000    0.667 -360.000    0.333 -180.000
1.0500   0.472 -94.244    0.472 -188.744    0.666  -1.058     0.335 -177.894

BRANCH LINE COUPLER CALCULATION
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     G-S11   G-S11    G-S12   G-S12     G-S13   G-S13     G-S14   G-S14
1.0000   0.000 -180.021   0.707 -90.000     0.707 -180.000    0.000 -269.979
1.0500   0.095 -96.591    0.695 -100.756    0.707 -190.913    0.094 -15.230
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     G-S21   G-S21    G-S22   G-S22     G-S23   G-S23     G-S24   G-S24
1.0000   0.707 -90.000    0.000 -180.021    0.000 -269.979    0.707 -180.000
1.0500   0.695 -100.756   0.095 -96.591     0.094 -15.230     0.707 -190.913
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     G-S31   G-S31    G-S32   G-S32     G-S33   G-S33     G-S34   G-S34
1.0000   0.707 -180.000   0.000 -269.979    0.000 -180.021    0.707 -90.000
1.0500   0.707 -190.913   0.094 -15.230     0.095 -96.591     0.695 -100.756
FREQ      MAG    PHASE     MAG    PHASE      MAG    PHASE      MAG    PHASE
 GHZ     G-S41   G-S41    G-S42   G-S42     G-S43   G-S43     G-S44   G-S44
1.0000   0.000 -269.979   0.707 -180.000    0.707 -90.000     0.000 -180.021
1.0500   0.094 -15.230    0.707 -190.913    0.695 -100.756    0.095 -96.591

ENTER COMMAND(LDT,ANL,CPT,LNL)
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Figure 4 — Parameters for Networks A, B, C, D, E, F, and G of Example